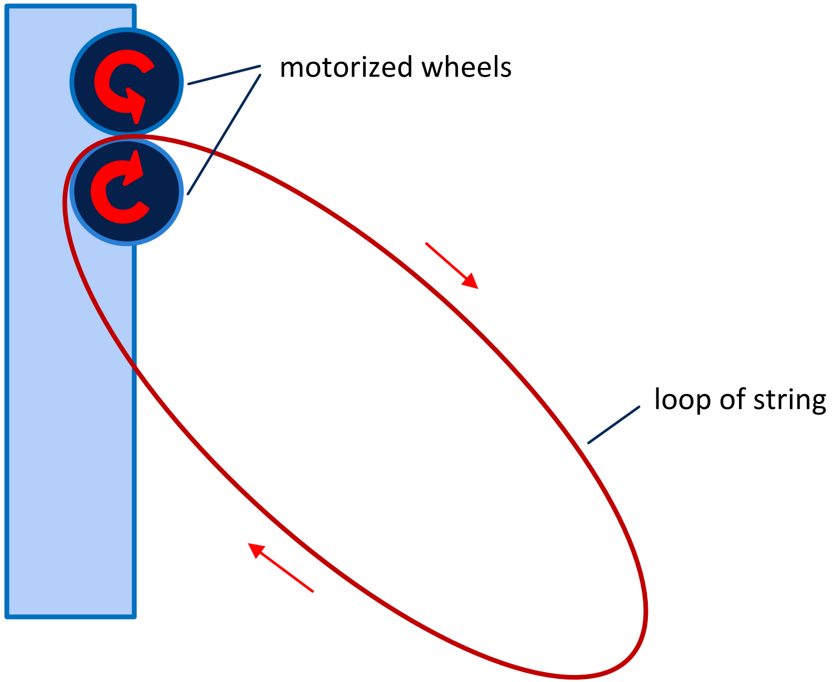
13 - STRING SHOOTER

FRANCE – UNIVERSITÉ DE LYON
REPORTER – PASCAL WANG

THE STRING SHOOTER



- A loop of string is set in motion by two rotating wheels

THE STRING SHOOTER

- A loop of string is set in motion by two rotating wheels
- Well-defined stationary shape

EXPLAIN THE
OVERALL **SHAPE** OF THE LOOP AND INVESTIGATE THE PROPAGATION OF **WAVES** ON THE STRING.

PART I: NAIVE ANALYSIS OF THE SHAPE

Part I: Naive analysis

- Control parameters (elasticity neglected for our strings)
  - String velocity $\boldsymbol{v}$
  - String length $\boldsymbol{L}$
  - Linear mass $\boldsymbol{\mu}$

    (weight: $\boldsymbol{\mu}\ \boldsymbol{g}\ \boldsymbol{L}$)

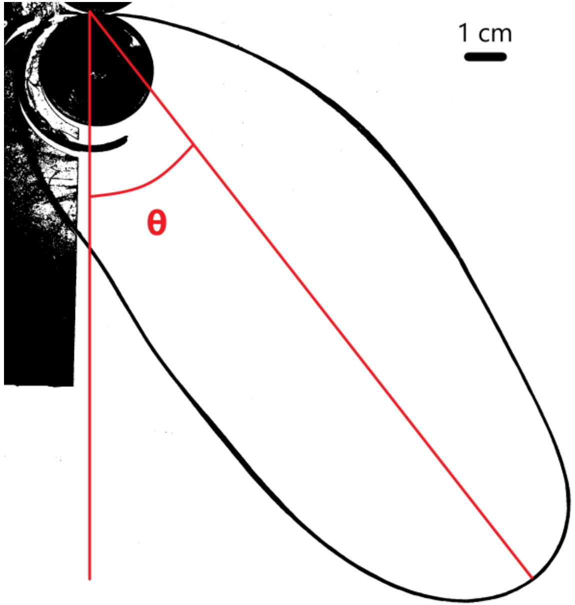
$\Rightarrow$

Dimensionless number comparing inertia and gravity

$$Fr^2 = \frac{E_{kin}}{E_{pot}} = \frac{\mu \cdot v^2}{\mu \cdot g \cdot L} = \frac{v^2}{g \cdot L}$$

$\boxed{\text{This inertia-gravity competition idea is too naive!}}$

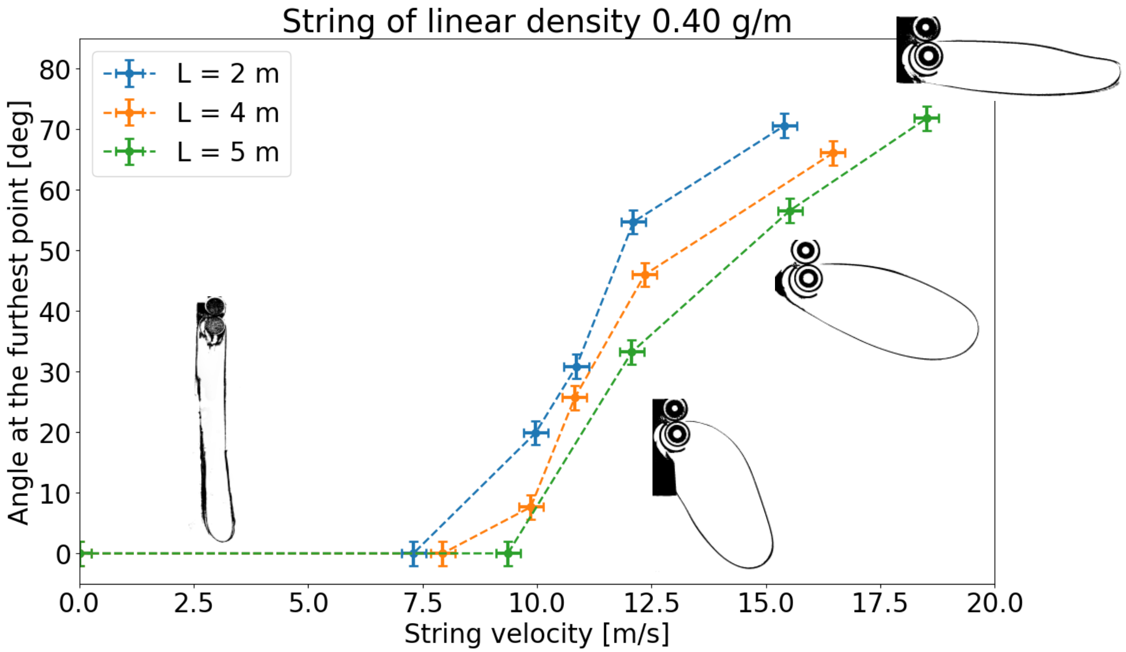
Part I: Definition



Definition of the angle at the furthest point

- Scalar quantity that allows quantitative comparison between shapes

Part I: Missing ingredient

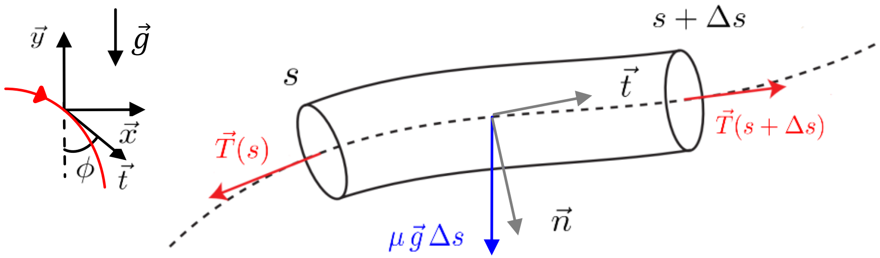


$$Fr^2 = \frac{v^2}{g \cdot L}$$

- Strong dependency on the velocity $\boldsymbol{v}$
- Clear onset of a phenomenon controlled by v and not Fr^2

Missing physical ingredient

Part I: Total disagreement

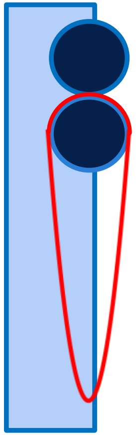


Momentum budget on a moving piece of string

$$\underbrace{\mu v^2 \frac{d\phi}{ds}\vec{n}}_{acceleration} = \underbrace{\frac{d}{ds}\left(T(s)\vec{t}\right)}_{tension} + \underbrace{\mu\vec{g}}_{gravity} \quad [1]$$

<u>Equation:</u> catenary with effective tension $T_{eff} = T - \mu v^2$

<u>Solution:</u> The shape depends only on geometrical conditions, not v, μ.

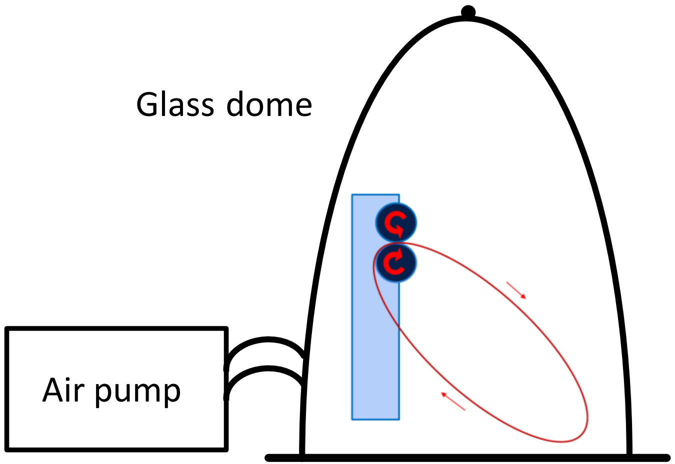


➡ total disagreement with experiment, a physical ingredient is missing!

[1] D. Young Strings, Chains, and Rope

PART II: DRAG IS THE SOLUTION

Part II: Drag matters



Sketch of the apparatus in vacuum chamber

Part II: Drag matters

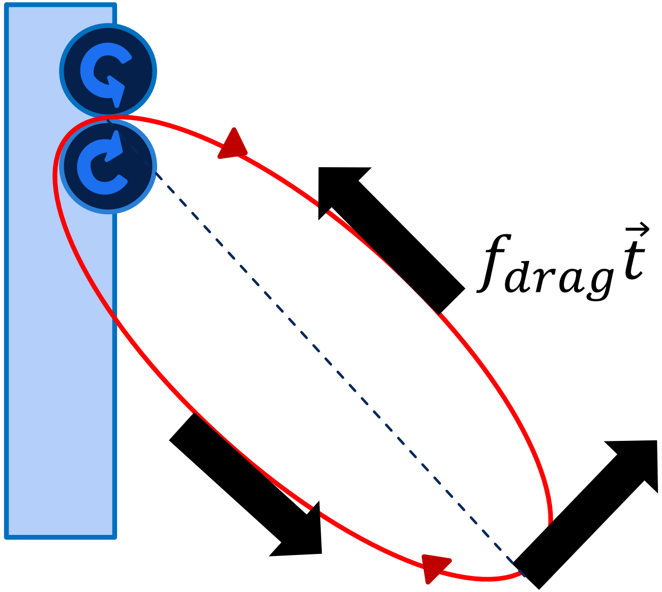
P = 1.0 bar

P = 0.85 bar

P = 0.7 bar

Experiment in a vacuum chamber (fixed velocity $\boldsymbol{v}$)

Part II: Drag creates torque



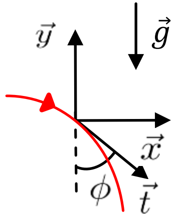
- NO LIFT on the string
- $\oint_C \overrightarrow{f_{drag}}(s)ds \simeq \vec{0}$

Drag induces torque Γ_{drag} and elevates the string!

Part II: Prediction of the shape

Recall the equation from the preliminary analysis:

$$\underbrace{\mu v^2 \frac{d\phi}{ds}\vec{n}}_{acceleration} = \underbrace{\frac{d}{ds}\left(T(s)\vec{t}\right)}_{tension} + \underbrace{\mu\vec{g}}_{gravity}$$



The resulting equation on ϕ:

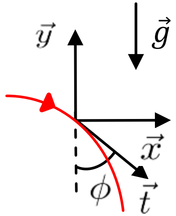
$$\frac{d\phi}{ds} = const \cdot \frac{\sin^2\phi}{}$$

(catenary equation)

Part II: The shape equation

Adding constant drag (per unit length) to the equation:

$$\underbrace{\mu v^2 \frac{d\phi}{ds}\vec{n}}_{acceleration} = \underbrace{\frac{d}{ds}\left(T(s)\vec{t}\right)}_{tension} + \underbrace{\mu\vec{g}}_{gravity} \underbrace{- f_{drag}\vec{t}}_{drag}$$



The resulting equation on ϕ:

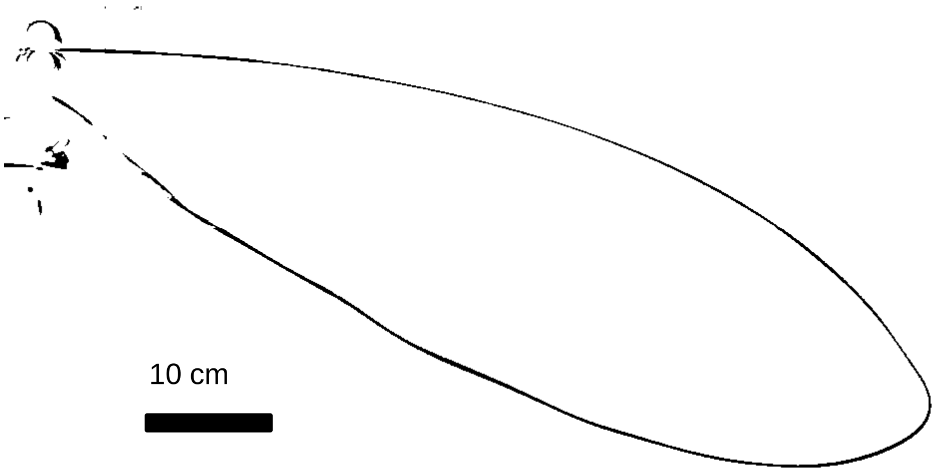
$$\frac{d\phi}{ds} = const \cdot \frac{\sin^2\phi}{\tan^D \frac{\phi}{2}}$$

$$\boxed{D = \frac{f_{drag}}{\mu g} = \frac{drag\ per\ unit\ length}{weight\ per\ unit\ length}}$$

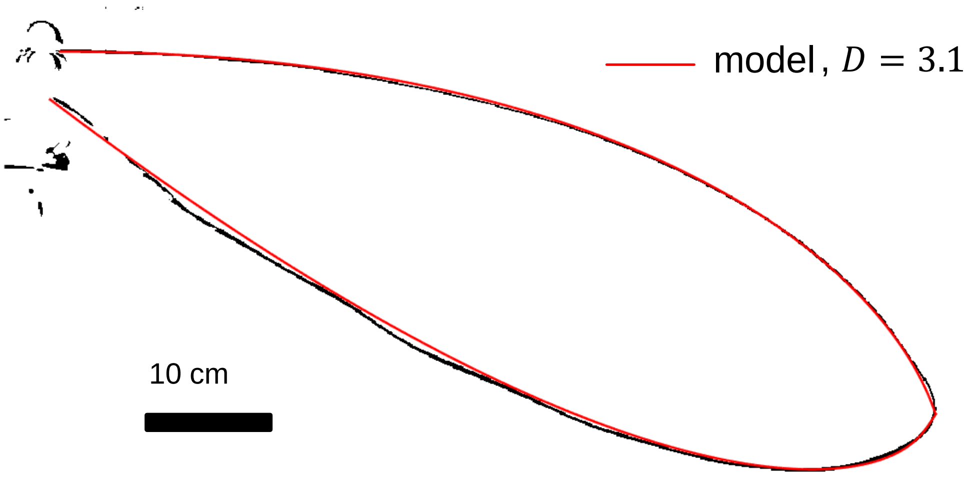
Dimensionless number of the problem!

[1] A. Dowling, J. Fluid Mech.187, 507 (1988).
[2] P. Williams, D. Sgarioto, and P. M. Trivailo, Aerospace Sci. Technol.12, 347 (2008).

Part II: Shape prediction

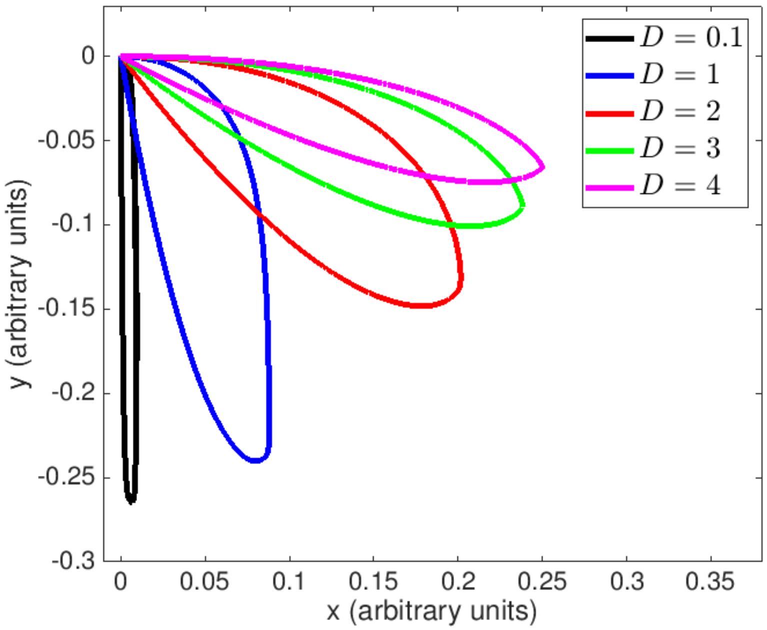


Part II: Shape prediction



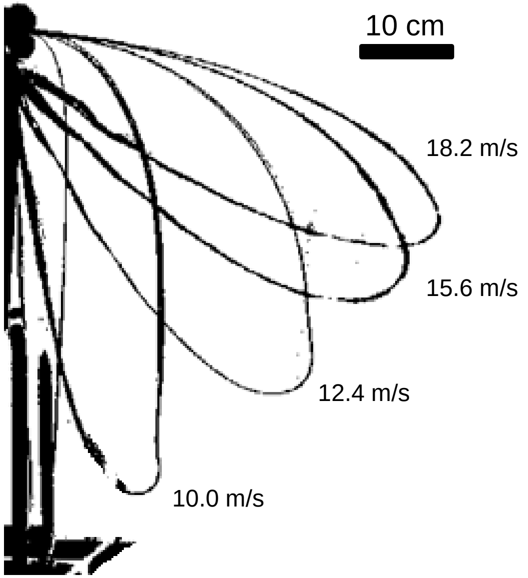
The value of D predicts the shape very accurately

Part II: Shape prediction



Model

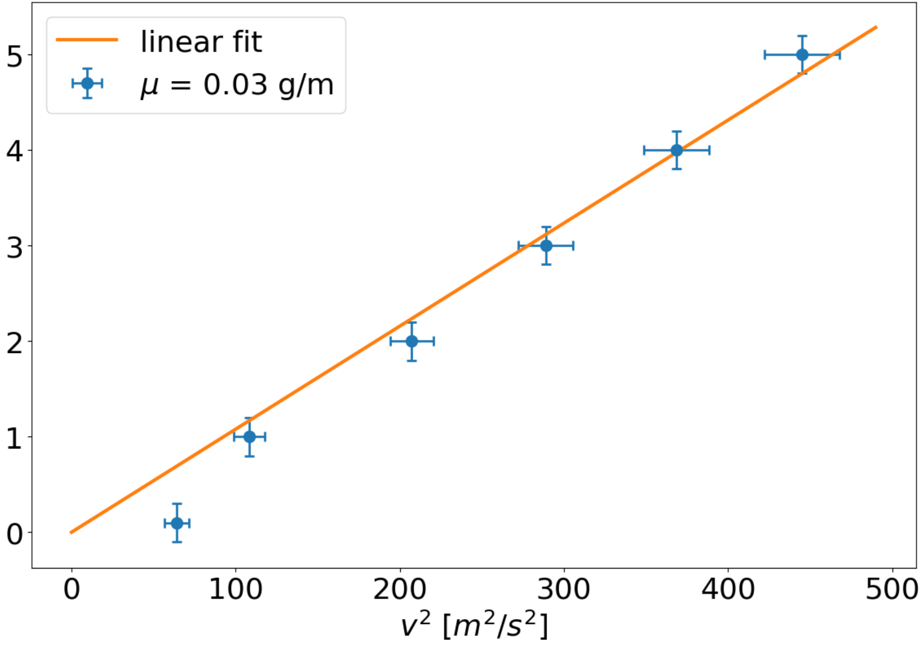
$$D = \frac{f_{drag}}{\mu g}$$



Experimental shapes (fixed μ)

Part II: Drag measurement

$$D = \frac{f_{drag}}{\mu g}$$



When the string rises, $R_e \sim 10^3$

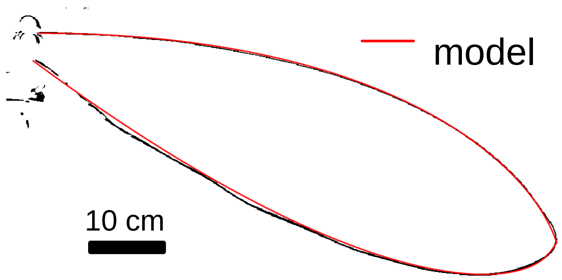
↓ [1]

$$f_{drag} = \frac{1}{2} C_D (2\pi R) \rho \cdot \boldsymbol{v^2}$$

[1] Slender structures and axial flow , M. P. Paidoussis

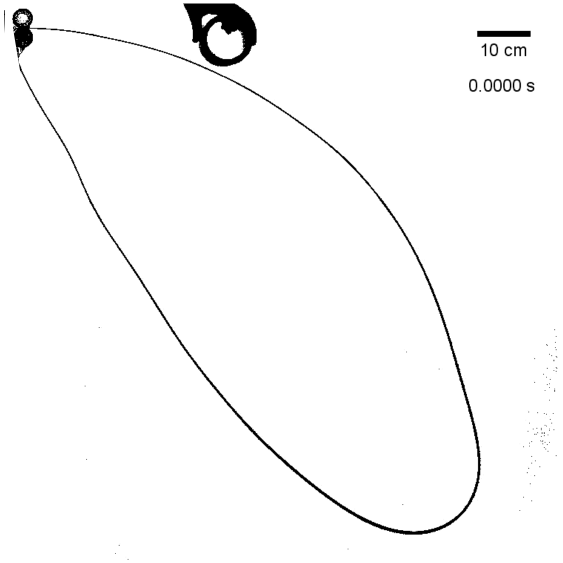
Recap

- <u>Stationary shape:</u>
  - ✓ drag creates torque and elevates the string
  - ✓ parameter of the problem $D = \frac{f_{drag}}{\mu g}$
  - ✓ equation and prediction of stationary shape



PART 3. WAVE PROPAGATION

Part III: Striking observations



Wave propagation in a moving string

3 observations:

1) Tapping at the top creates **two waves**

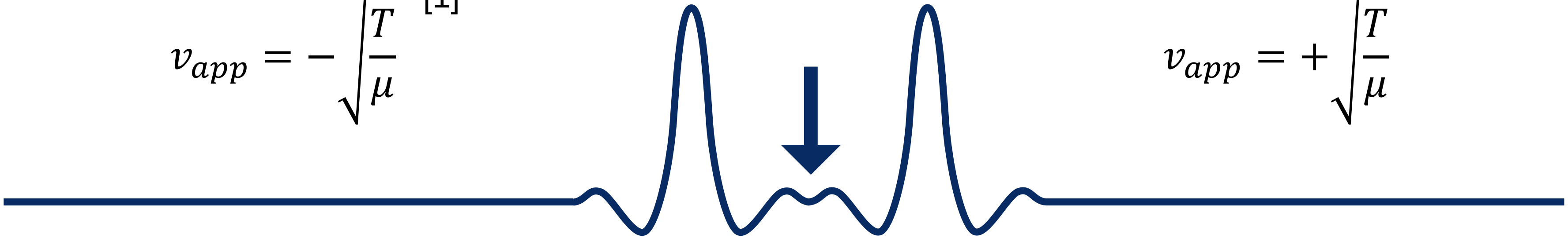
2) Waves **slow down** and **die** at the turning point

3) Top wave propagates **downstream**, bottom wave goes **upstream**

Part III: Slow and fast waves

$$v_{app} = -\sqrt{\frac{T}{\mu}}$$ [1]

$$v_{app} = +\sqrt{\frac{T}{\mu}}$$



$$\boldsymbol{v_{string} = 0}$$

slow wave

$$\boldsymbol{v_{app} = v_{string} - \sqrt{\frac{T}{\mu}}}$$

fast wave

$$\boldsymbol{v_{app} = v_{string} + \sqrt{\frac{T}{\mu}}}$$

$$\boldsymbol{v_{string}}$$

$$\boldsymbol{v_{string} \neq 0}$$

[1] The Feynman Lectures on Physics

Part III: Why two waves?

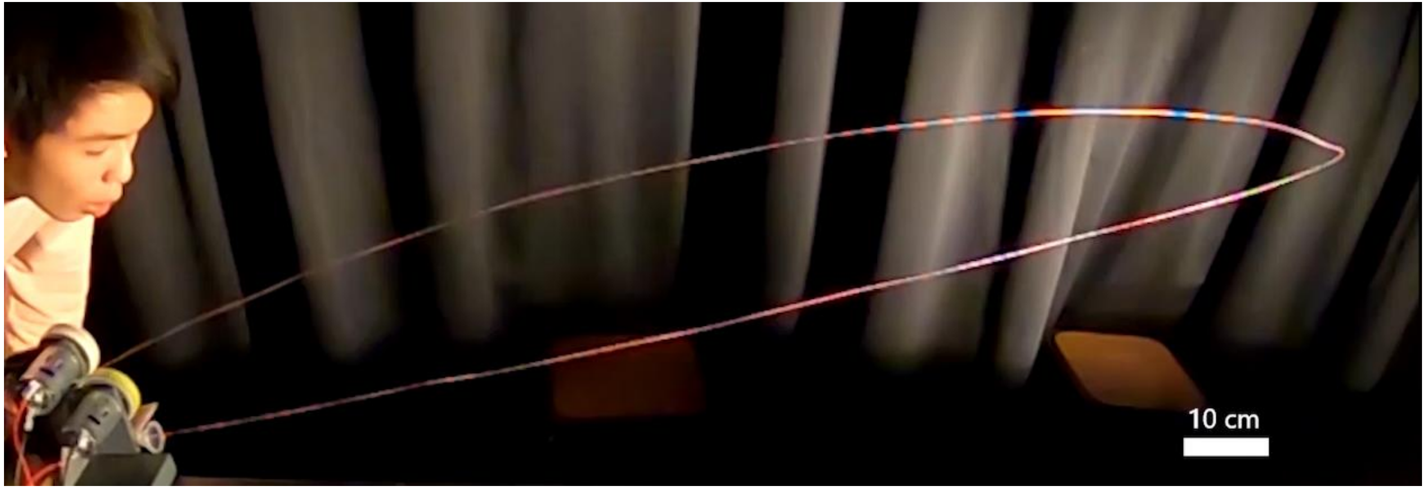
<u>Observation 1:</u> Tapping at the top creates a wave at the bottom

The bottom wave is born from a reflection of the fast wave (created at the top) between the wheels!

Slow motion (0.33x) video of the propagation of waves

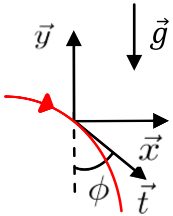
Part III: Waves die at the tip

Observation 2: Slow waves slow down and die as they reach the tip



Part III: Waves die at the tip

<u>Observation 2:</u> Slow waves slow down as they reach the tip



(1) Momentum equation on $\vec{n}$

$$T(s) = \mu v_{string}^2 + \frac{\mu g \sin(\phi)}{\frac{d\phi}{ds}}$$

(2) Velocity addition

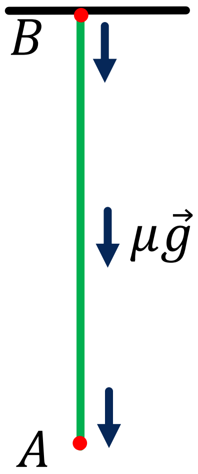
$$v_{app} = v_{string} - \sqrt{\frac{T}{\mu}}$$

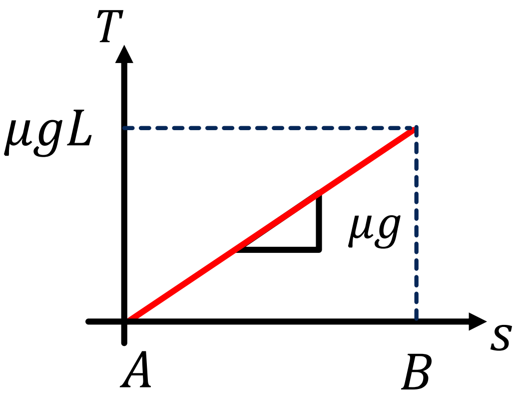
When the string is vertical: $\phi = 0 \overset{(1)}{\Longrightarrow} T = \mu v_{string}^2 \overset{(2)}{\Longrightarrow} \boxed{v_{app} = 0}$

The slow waves die when the string is vertical!

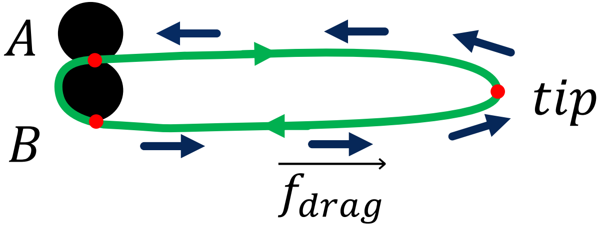
Part III: Limit cases

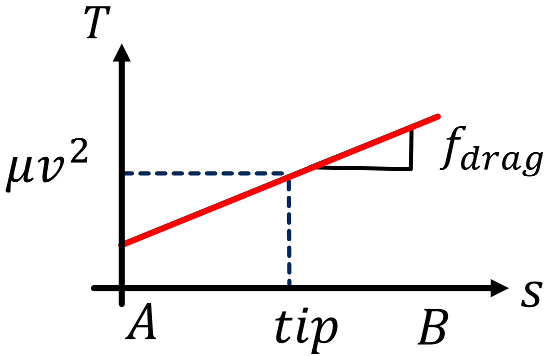
Hanging rope $D = 0$:





Friction dominated $D \gg 1$:





$$T(s) - \mu v^2 \propto (s - s_{tip})$$

Part III: Waves die at the tip

$$\begin{cases} T(s) - \mu v^2 \propto (s - s_{tip}), \; D \gg 1 \\ v_{app} = v - \sqrt{\dfrac{T(s)}{\mu}} \end{cases}$$

$$\downarrow$$

$$v_{app} = \dot{s} \propto (s - s_{tip})$$

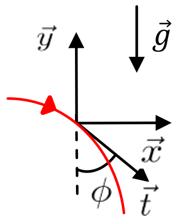
for small T

$$\downarrow$$

$s(t)$ and $v_{app}(t)$ relax exponentially

Part III: Upstream, downstream

<u>Observation 3:</u> Slow waves at the top are downstream, slow waves at the bottom are upstream



(1) Momentum equation on $\vec{n}$

$$T(s) = \mu v_{string}^2 + \frac{\mu g \sin(\phi)}{\frac{d\phi}{ds}}$$

(2) Velocity addition

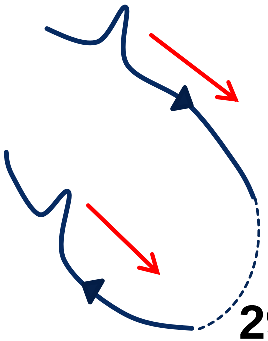
$$v_{app} = v_{string} - \sqrt{\frac{T}{\mu}}$$

At the top: $0 < \phi < \frac{\pi}{2} \overset{(1)}{\Rightarrow} T(s) < \mu v_{string}^2 \overset{(2)}{\Rightarrow} \boxed{v_{app} > 0}$

Waves at the top are downstream

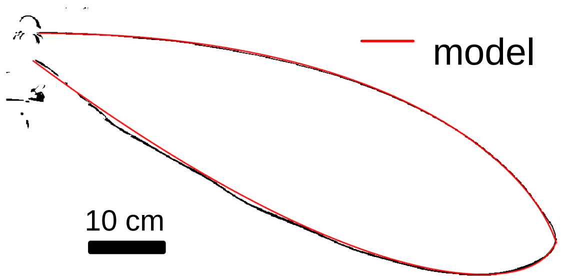
At the bottom: $-\frac{\pi}{2} < \phi < 0 \overset{(1)}{\Rightarrow} T(s) > \mu v_{string}^2 \overset{(2)}{\Rightarrow} \boxed{v_{app} < 0}$

Waves at the bottom are upstream



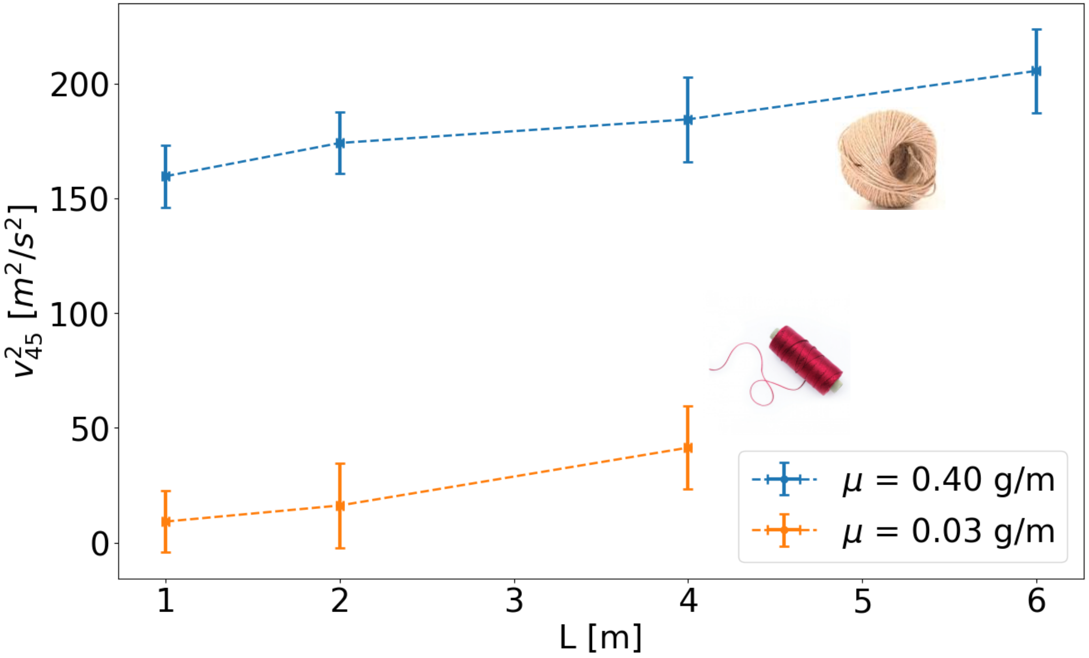
Solution summary

- Stationary shape:
  - ✓ drag creates torque and elevates the string
  - ✓ parameter of the problem $D = \frac{f_{drag}}{\mu g}$
  - ✓ equation and prediction of stationary shape



- Wave propagation:
  - ✓ tapping at the top creates a second wave born from reflection
  - ✓ upstream (bottom) and downstream (top) waves
  - ✓ slow waves slow down exponentially as they die out at tip

Linear mass matters

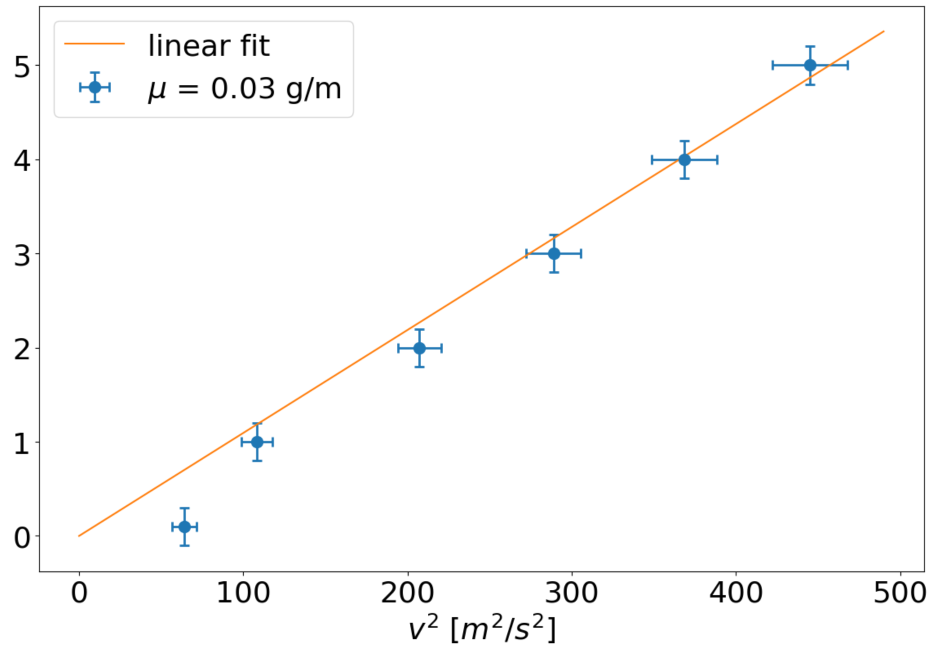


Linear mass $\boldsymbol{\mu}$ matters!

Governing parameter has to include $\boldsymbol{\mu}$.

Drag measurement

$$D = \frac{f_{drag}}{\mu g}$$



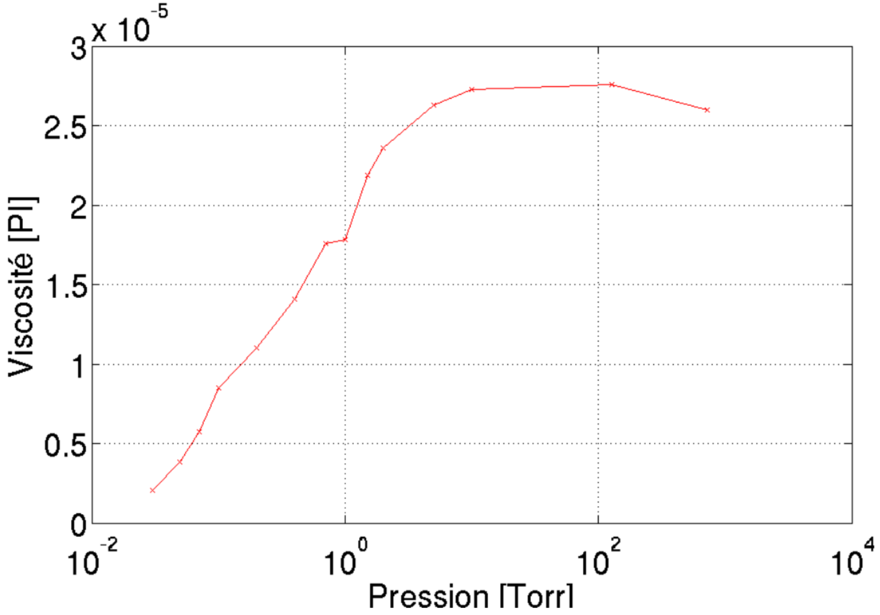
When the string rises, $R_e \sim 10^3$

$$f_{drag} = \frac{1}{2} C_D (2\pi R)\rho \cdot \boldsymbol{v^2}$$

Drag coefficient measurement

$$C_D = 0.011 \pm 0.001$$

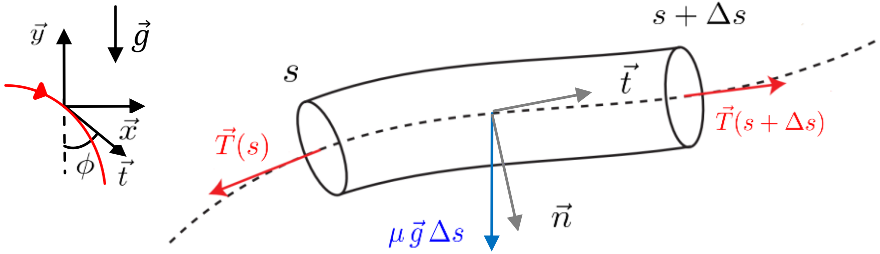
Air viscosity



Viscosity as a function of pressure [1]
($1\ Torr = 133.322\ Pa$)

[1] Viscosité de l’air, Matthieu Schaller and Xavier Buffat

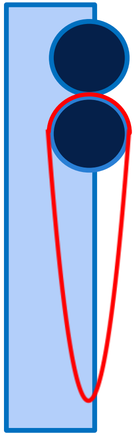
Stationary shape



Momentum budget on a moving piece of string

$$\underbrace{\mu v^2 \frac{d\phi}{ds}\vec{n}}_{acceleration} = \underbrace{\frac{d}{ds}\left(T(s)\vec{t}\right)}_{tension} + \underbrace{\mu\vec{g}}_{gravity}$$

$$\vec{0} = \frac{d}{ds}(T - \mu v^2)\vec{t} + \mu\vec{g}$$

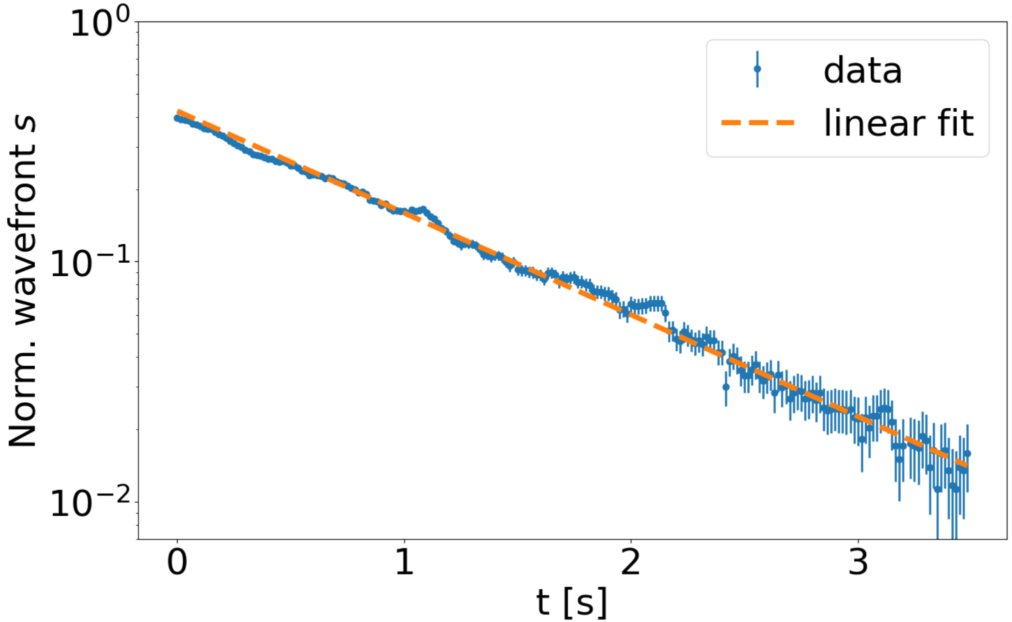


<u>Solution</u> : catenary with effective tension $T_{eff} = T - \mu v^2$

The shape depends only on geometrical conditions, not v, μ.

➡ total disagreement with experience, a physical ingredient is missing !

EXPONENTIAL DECREASE OF THE WAVE



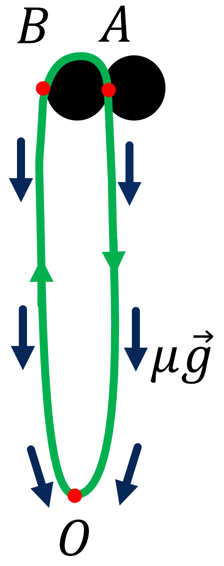
$$\frac{s_{wf} - s_{tip}}{s_{wheel} - s_O} = e^{-t/\tau}$$

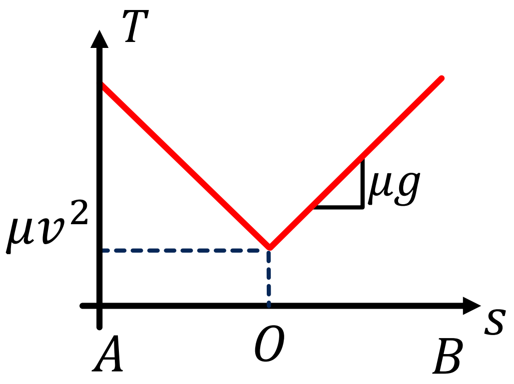
$$\tau = \frac{2\mu v}{f_{drag}}$$

$$\tau = 1.03\ s$$

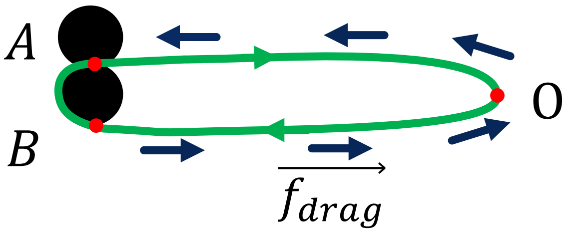
Part III: Limit cases

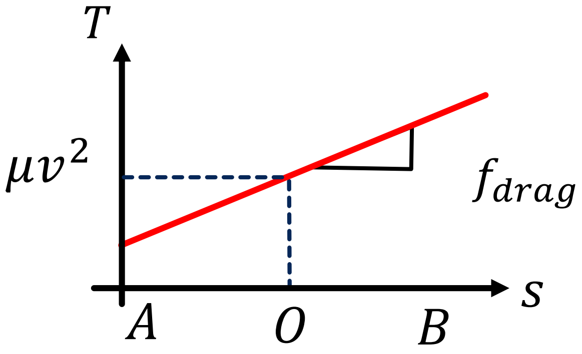
Gravity dominated $D \ll 1$:





Friction dominated $D \gg 1$:





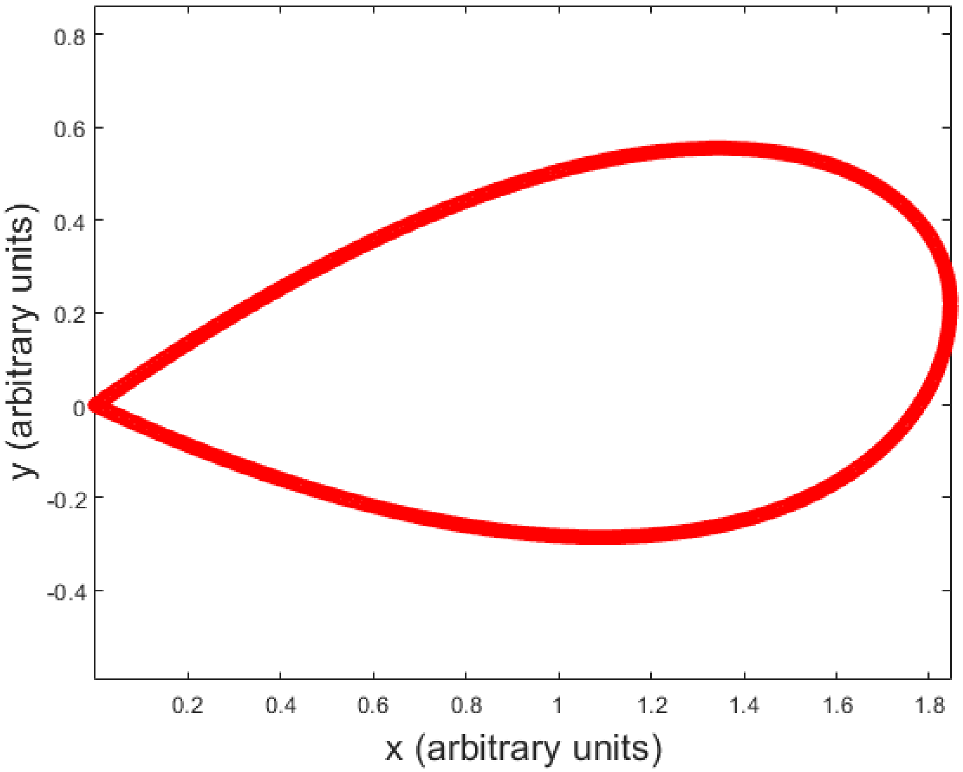
$$T(s) - \mu v^2 \propto (s - s_{tip})$$

ANNEX

- Reynolds number estimation

$$Re = \frac{v_{string} \cdot d_{string}}{\nu_{air}} \sim \frac{10 \cdot 10^{-3}}{10^{-5}} \sim 10^3$$

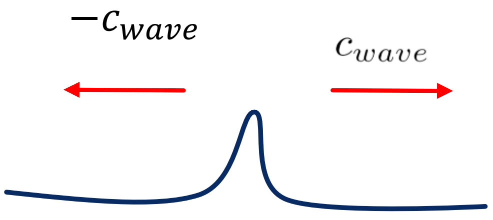
ANNEX



Tilted at 35° angle

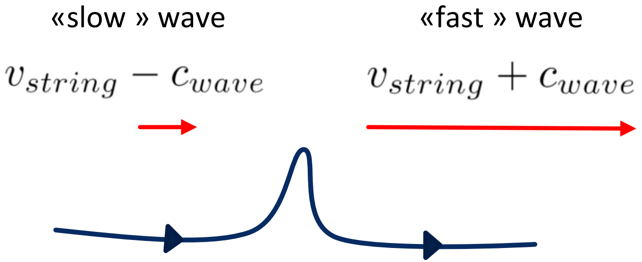
Wave propagation

String at rest



$$v_{app} = \pm c_{wave}$$

String in motion



$$v_{app} = v_{string} \pm c_{wave}$$

Velocity addition